Significant Figures and Uncertainties **Chemistry BC3252y**

The topic of significant figures is taught in General Chemistry, so I expect that answers on problem sets and exams will always be expressed with the correct number of significant figures. The notes below are for review.

Significant figures. For any measured quantity, the rightmost digit is implied to be uncertain.

22.563 is uncertain in the thousandths place. This has **five** significant figures.

Uncertainties: A more informative way of expressing the uncertainty of a quantity **x** is to give its **absolute uncertainty, Δx**. For 15.9994 ± 0.0003, the absolute uncertainty is ± 0.0003. Absolute uncertainties have the same units as the values with which they are associated. The value is written to the same decimal place as the (usually) single non-zero digit of the uncertainty. The **relative uncertainty**, Δx/x, is also sometimes useful. It has no units.

Please note: in problems and exams, you should assume that all data and constants are given with the correct number of significant figures. Some fundamental constants have uncertainties, and therefore carry a finite number of significant figures. Example: Avogadro's number, N_A, is $6.0221415(10) \times 10^{23}$. The (10) expresses the fact that the rightmost digits are uncertain by this quantity, so this has seven significant figures. Atomic weights have uncertainties that can be found in good periodic charts. Example: the atomic weight of O is 15.9994 ± 0.0003, so it has six significant figures.

Some quantities are *exact*: these have an infinite number of significant figures. There are exactly two O's in H_2O. By definition, there are exactly 4.184 Joules per calorie. 0°C = 273.15 K, exactly. If a problem says 25°C, treat that as exact. If a problem says *one* atmosphere or *two* moles, treat those as exact.

In solving problems, you need to know how significant figures combine.

Rule I. When you **add or subtract,** the result should be written to the same **decimal place** (hundreds, tenths, etc.) as the term with the fewest **decimal places.**

22.513 + 0.43 = 22.943 before considering sig. figs.

Since the 0.43 could be off in the hundredths place (e.g. by 0.02),
the answer should be rounded to the hundredths place:

$22.513 + 0.43 = 22.943 \rightarrow 22.94$

In answers to problem sets, I often write the unrounded value first, then an arrow (→) followed by the correctly rounded value. I also underline the rightmost sig. fig. in intermediate steps.

Rule II. When you **multiply or divide,** the result has the same number of **significant figures** as the term with the fewest **significant figures.**

(2.43 L)(1.4667 atm) = 35.64081 L-atm → 35.6 L-atm

In combined equations, apply these rules step by step:

$23.679 + 0.00478(50.30) = 23.67\underline{9} + 0.24\underline{0}434 = 23.91\underline{9}434 \rightarrow 23.919$

even though 0.00478 has three sig. figs., the answer has five, because of addition.

Unrounded values should be used in subsequent calculations to avoid roundoff error.

How do significant figures work through functions?

I. The easiest way to answer this for a particular function is to use you calculator.

Suppose z = 2.34 x 10^6, what is log z? log (2.34 x 10^6) = 6.3692159.. [before rounding]

Note that **log z** always means the base-10 logarithm; **ln z** the base-e.

Add (or subtract) an arbitrary amount, say 2, from the rightmost digit:

log (2.36 x 10^6) = 6.37<u>2</u>9120…

6.37<u>2</u>9120.. – 6.3692159.. = 0.0037 These differ in the <u>third decimal place</u>.

Therefore, correctly rounded, log (2.34 x 10^6) = 6.369

This illustrates a **general rule for logs**: the number of significant figures in the argument equals the number of digits to the right of the decimal place in the (base-10) logarithm. (This is because the other part of the log, that to the left of the decimal, conveys the power of ten.) The part of a logarithm to the right of the decimal is called the mantissa, the part to the left is the characteristic. (I do not expect you to know these terms.)

Reverse: pH = 4.52 → $[H^+]$ = 3.0 x 10^{-5}

II. A more sophisticated answer is found using calculus.

Consider how an uncertainty in x, Δx, results in an uncertainty in f, Δf, when f = f(x).

If Δx is small, then df/dx = Δf/Δx. Thus Δf = (df/dx) Δx

Thus **for a function f(x), the uncertainty in f, Δf, is given by Δf = (df/dx) Δx.**

Apply this first to **ln z:** d (ln z)/dz = 1/z. Therefore Δ ln z = Δz/z

This is an interesting and useful result: **the relative uncertainty of x, (Δx/x) is equal to the absolute uncertainty of its natural logarithm.**

What about **log x**?

First, recall how ln is related to log. If y = log x then x = 10^y.

Take ln of both sides: ln x = ln(10^y) = y ln 10. Thus **ln x = (ln 10) (log x)**

This should look familiar if we write **ln x = 2.303 log x** where 2.303 = ln 10.

So log x = ln x/(ln 10) = ln x/2.303

d (log x)/dx = [d (ln x)/dx]/2.303 = 1/(2.303 x). Therefore Δ log x = (Δx/x) /2.303

Illustrate with the same numbers used above: if x = 2.34 x 10^6 and Δx = 0.02 x 10^6,

then Δx/x = 0.0085 so Δ log x = 0.0037.

The identical answer was found using the calculator method.