

**Free particle** of mass  $m$  moving in one-dimension  $x$ :  $V(x) = 0$

$$\psi(x) = A \sin(kx + \delta) \quad (\text{not normalized})$$

$$k^2 = 2mE, \text{ no quantization of energy}$$

**Particle in a one-dimensional box:** mass  $m$ ,  $V(x) = 0$  when  $0 < x < L$ ,  $V(x) = \infty$  otherwise

$$E_n = n^2 h^2 / 8mL^2 \quad n = 1, 2, 3 \dots \infty$$

$$\psi_n(x) = (2/L)^{1/2} \sin(n\pi x/L)$$

**Simple Harmonic Oscillator:** particle of mass  $m$  with harmonic force constant  $k$

$$V(x) = \frac{1}{2} kx^2 = \frac{1}{2} m\omega^2 x^2 \quad \text{where } \omega^2 = k/m \quad \text{Reduced variable } \xi = x/(\hbar/m\omega)^{1/2}$$

$$E_v = (v + 1/2) \hbar\omega \quad v = 0, 1, 2, 3 \dots \infty$$

$$\psi_v(\xi) = N_v H_v(\xi) e^{-\xi^2/2} \quad N_v = (2^v v! \pi^{1/2})^{-1/2}$$

**Hermite Polynomials  $H_v(\xi)$**

<b>v</b>	0	1	2	3
<b><math>H_v(\xi)</math></b>	1	$2\xi$	$4\xi^2 - 2$	$8\xi^3 - 12\xi$

**Particle on a Ring:** Particle mass  $m$ , ring of radius  $r$ , no torques:  $V(\varphi) = 0$

$$E_M = \hbar^2 M^2 / 2I \quad \text{where } I = mr^2 \quad M = 0, \pm 1, \pm 2, \pm 3 \dots \pm \infty$$

$$\Phi_M(\varphi) = (2\pi)^{-1/2} \exp(iM\varphi)$$

**Rigid rotor:** particle on mass  $m$  moving on a sphere of radius  $r$ :  $V(\theta, \varphi) = 0$

$$E_J = (\hbar^2 / 2I r^2) J(J+1) = B J(J+1) \quad J = 0, 1, 2 \dots \infty ; M = 0, \pm 1, \pm 2, \pm 3 \dots \pm J$$

$$\psi_{J,M}(\theta, \varphi) = Y_{J,M}(\theta, \varphi) = \Theta_{J,M}(\theta) \Phi_M(\varphi) \quad Y_{J,M}(\theta, \varphi) \text{ called Spherical Harmonics}$$

$$\Theta_{J,M}(\theta) = N_{J,M} P_{J,M}(\cos \theta) \quad \Phi_M(\varphi) = (2\pi)^{-1/2} \exp(iM\varphi)$$

**Associated Legendre Polynomials  $P_{J,M}(\cos \theta)$**  *(functions of  $\cos \theta$ )*

<b>J</b>	0	1	1	2	2	2
<b>M</b>	0	0	$\pm 1$	0	$\pm 1$	$\pm 2$
<b><math>P_{J,M}(\cos\theta)</math></b>	1	$\cos\theta$	$\sin\theta$	$\frac{1}{2} (3 \cos^2\theta - 1)$	$\sin\theta \cos\theta$	$\sin^2\theta$
<b><math>N_{J,M}</math></b>	$\sqrt{(1/2)}$	$\sqrt{(3/2)}$	$\sqrt{(3/4)}$	$\sqrt{(5/8)}$	$\sqrt{(15/4)}$	$\sqrt{(15/16)}$

**Hydrogenlike atom** (nucleus with charge  $Z$ , one electron with mass  $m_e$ )

$$E_n = -m_e (Ze^2 / 4\pi\epsilon_0)^2 / 2\hbar^2 n^2 = -Z^2 R_\infty / n^2 \quad (\text{identical to Bohr result})$$

three quantum numbers  $(n, \ell, m)$  where  $n = 1, 2, 3 \dots \infty$ ;  $0 \leq \ell < n$ ;  $|m| \leq \ell$

$$\psi_{n, \ell, m}(r, \theta, \varphi) = R_{n, \ell}(r) Y_{\ell, m}(\theta, \varphi) \quad Y \text{ same as above, with } (J, M) \rightarrow (\ell, m)$$

<b>n</b>	<b><math>\ell</math></b>	<b>Normalized Radial Functions <math>R_{n, \ell}(r)</math></b>
1	0	$2 (Z/a_0)^{3/2} \exp(-Zr/a_0)$
2	0	$(1/2\sqrt{2}) (Z/a_0)^{3/2} (2 - Zr/a_0) \exp(-Zr/2a_0)$
2	1	$(1/2\sqrt{6}) (Z/a_0)^{3/2} (Zr/a_0) \exp(-Zr/2a_0)$
3	0	$(2/81\sqrt{3}) (Z/a_0)^{3/2} [27 - 18(Zr/a_0) + 2(Zr/a_0)^2] \exp(-Zr/3a_0)$
3	1	$(4/81\sqrt{6}) (Z/a_0)^{3/2} [6(Zr/a_0) - Zr/a_0]^2 \exp(-Zr/3a_0)$
3	2	$(4/81\sqrt{30}) (Z/a_0)^{3/2} (Zr/a_0)^2 \exp(-Zr/3a_0)$