USEFUL MATH FACTS **Chemistry BC3253x**

SERIES

1. Taylor's series: $f(x) = f(a) + (x-a) f'(a) + (x-a)^2 f''(a)/2 + \ldots + (x-a)^n f^{(n)}(a)/n! + \ldots$
2. $e^x = 1 + x + x^2/2 + x^3/3! + \ldots$
3. $\sin x = x - x^3/3! + x^5/5! - x^7/7! + \ldots$
4. $\cos x = 1 - x^2/2! + x^4/4! - x^6/6! + \ldots$
5. $\ln(1 + x) = x - x^2/2 + x^3/3 - x^4/4 + \ldots$ $\quad -1 < x \leq 1$
6. $(1 + x)^{1/2} = 1 + x/2 - x^2/8 + x^3/16 - 5x^4/128 + \ldots$
7. $(1 \pm x)^{-1} = 1 \mp x + x^2 \mp x^3 + \ldots$ $\quad x^2 < 1$
8. $(1 \pm x)^{-2} = 1 \mp 2x + 3x^2 \mp 4x^3 + \ldots$ $\quad x^2 < 1$

TRIGONOMETRIC IDENTITIES

$\sin^2\theta + \cos^2\theta = 1$ $\qquad \tan\theta = \sin\theta / \cos\theta$

$\sin\alpha \sin\beta = ½ [\cos(\alpha-\beta) - \cos(\alpha+\beta)]$ $\qquad \sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$

$\cos\alpha \cos\beta = ½ [\cos(\alpha-\beta) + \cos(\alpha+\beta)]$ $\qquad \cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$

$\sin\alpha \cos\beta = ½ [\sin(\alpha+\beta) + \sin(\alpha-\beta)]$ $\qquad \sin 2\theta = 2 \sin\theta \cos\theta$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$e^{i\theta} = \cos\theta + i \sin\theta$ $\qquad \sin\theta = (e^{i\theta} - e^{-i\theta}) / 2i$ $\qquad \cos\theta = (e^{i\theta} + e^{-i\theta}) / 2$

DEFINITE INTEGRALS

(m,n are integers)

1. $\int_0^\infty x^n e^{-ax}\, dx = \dfrac{n!}{a^{n+1}}$ $\quad a > 0$ and $n \geq 0$
2. $\int_0^\infty e^{-ax^2}\, dx = \dfrac{1}{2}\left(\dfrac{\pi}{a}\right)^{1/2}$
3. $\int_0^\infty x\, e^{-ax^2}\, dx = \dfrac{a}{2}$
4. $\int_0^\infty x^{2n} e^{-ax^2}\, dx = \dfrac{1 \cdot 3 \cdot 5 \cdot \cdot (2n-1)}{2^{n+1}}\left(\dfrac{\pi}{a^{2n+1}}\right)^{1/2}$ $\quad n \geq 1$
5. $\int_0^\infty x^{2n+1} e^{-ax^2}\, dx = \dfrac{n!}{2a^{n+1}}$
6. $\int_0^\infty \cos kx\, e^{-ax^2}\, dx = \dfrac{1}{2}\left(\dfrac{\pi}{a}\right)^{1/2} e^{-k^2/4a}$
7. $\int_0^a \sin\left(\dfrac{n\pi x}{a}\right) \sin\left(\dfrac{m\pi x}{a}\right) dx = \int_0^a \cos\left(\dfrac{n\pi x}{a}\right) \cos\left(\dfrac{m\pi x}{a}\right) dx = \dfrac{a}{2}\delta_{mn}$ $\quad m, n > 0$

INDEFINITE INTEGRALS

(m and n are integers)

1. $\int \sin^2 ax\,dx = \frac{x}{2} - \frac{1}{4a}\sin 2ax$

2. $\int \cos^2 ax\,dx = \frac{x}{2} + \frac{1}{4a}\sin 2ax$

3. $\int x \sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$

4. $\int x \cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$

5. $\int x^2 \sin ax\,dx = \frac{2x}{a^2}\sin ax - \frac{a^2x^2 - 2}{a^3}\cos ax$

6. $\int x^2 \cos ax\,dx = \frac{2x}{a^2}\cos ax + \frac{a^2x^2 - 2}{a^3}\sin ax$

7. $\int x \sin^2 ax\,dx = \frac{x^2}{4} - \frac{x}{4a}\sin 2ax - \frac{1}{8a^2}\cos 2ax$

8. $\int x \cos^2 ax\,dx = \frac{x^2}{4} + \frac{x}{4a}\sin 2ax + \frac{1}{8a^2}\cos 2ax$

9. $\int x^2 \sin^2 ax\,dx = \frac{x^3}{6} - \left\{\frac{x^2}{4a} - \frac{1}{8a^3}\right\}\sin 2ax - \frac{x}{4a^2}\cos 2ax$

10. $\int x^2 \cos^2 ax\,dx = \frac{x^3}{6} + \left\{\frac{x^2}{4a} - \frac{1}{8a^3}\right\}\sin 2ax + \frac{x}{4a^2}\cos 2ax$

11. $\int \sin ax \cos ax\,dx = \frac{1}{2a}\sin^2 ax$

12. $\int \sin^2 ax \cos^2 ax\,dx = \frac{x}{8} - \frac{1}{32a}\sin 4ax$

13. $\int \sin mx \sin nx\,dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)}$ $\qquad m^2 \neq n^2$

14. $\int \cos mx \cos nx\,dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)}$ $\qquad m^2 \neq n^2$

15 $\int \sin mx \cos nx\,dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)}$ $\qquad m^2 \neq n^2$

16. $\int e^{ax}dx = \frac{e^{ax}}{a}$

17. $\int x\,e^{ax}dx = \frac{e^{ax}}{a^2}(ax - 1)$

18. $\int x^2\,e^{ax}dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$

19. $\int x^m\,e^{ax}dx = \frac{x^m e^{ax}}{a} - \frac{m}{a}\int x^{m-1}\,e^{ax}dx$