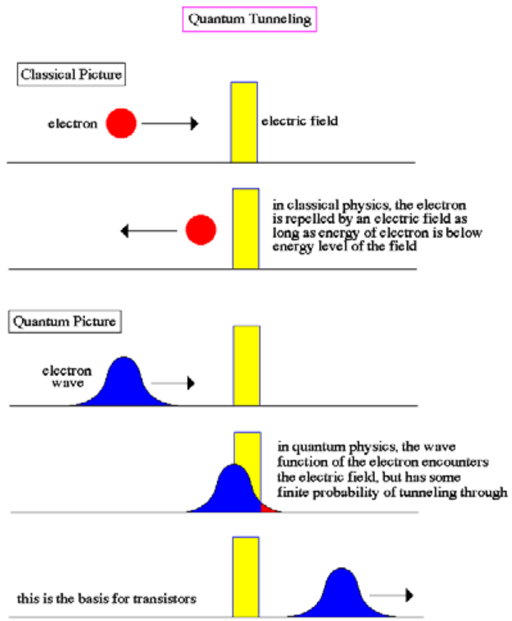
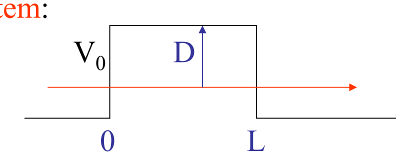
Tunneling



Tunneling

SHO and H-atom wavefunctions both extend into classically forbidden regions. Any physical consequences? not really…

But consider the following system:



I. $V = 0$ for $x < 0$

II. $V = V_0$ for $0 < x < L$

III. $V = 0$ for $x > L$

Schrodinger equation (1D): $d^2\psi/dx^2 = -2m(E-V)/\hbar^2$

Let $k^2 = 2mE/\hbar^2$ k is real. Then

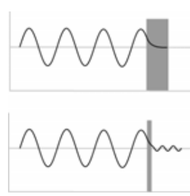
Possible solutions: $\psi_I(x) = A \sin(kx)$ and $\psi_{III}(x) = C \sin(kx)$

But suppose in region II that $E < V_0$. Let $D = V_0 - E$.

One solution in this region is $\psi_{II}(x) = B \exp(-k'x)$

where $k'^2 = 2m(V_0-E)/\hbar^2 = 2mD/\hbar^2$

Tunneling



$\psi_I(x) = A \sin(kx)$ and $\psi_{III}(x) = C \sin(kx)$

$\psi_{II}(x) = B \exp(-k'x)$ where $k'^2 = 2m(V_0-E)/\hbar^2$

At $x = 0$, ψ is continuous, so $B = A$.

$\psi_{II}(0) = A$, but $\psi_{II}(L) = A \exp(-k'x)$

So $\psi_{II}(L)/\psi_{II}(0) = \exp(-k'L)$

$\psi_{II}^2(L)/\psi_{II}^2(0) = \exp(-2k'L) =$ $\mathbf{\exp\{-2(2mD)^{1/2}L/\hbar\}}$

Probability that particle, starting on the left, emerges on right!

The bigger the { } the smaller the tunnelling:

decreases as mass m, depth thru barrier D, thickness L, increase.

Lighter particles will tunnel through thinner lower barriers.

This derivation a bit sketchy (should use time dependent wave packets; worry about phases) but result qualitatively correct.

Tunneling: applications

Radioactive Decay:

forces holding nucleons together very short-ranged;

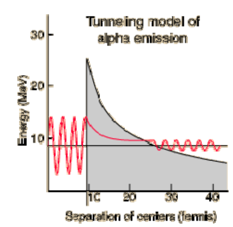
α-particle (He^{2+}) away from nucleus strongly repelled.

thin barrier keeps particle inside nucleus!

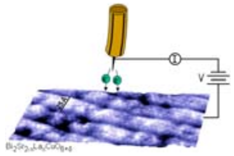
α-particle emission is a tunneling phenomenon:

governed by rules of probability

half-life depends on thickness and height of barrier.



Tunneling: applications



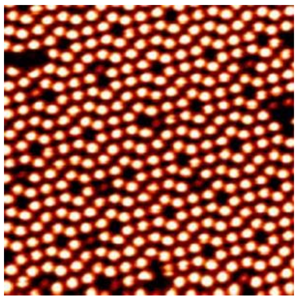
Scanning Tunneling Microscopy

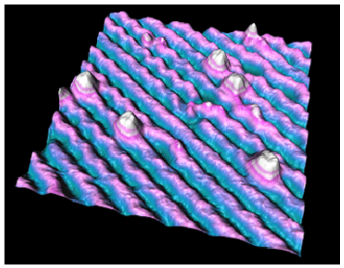
electrons tunnel between atom-scale tip and surface

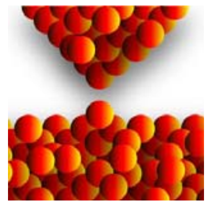
tunneling, so current flow falls <u>exponentially</u> with height

create image giving shape of surface on atomic scale!

Nobel Prize (Binnig-Rohrer 1986)







Tunneling: applications

Electron transport in proteins and DNA

through-space and through-bond processes

(work of Prof. Magyar and his group)

rate of transfer falls <u>exponentially with distance</u>:

hallmark of a tunneling process!

Non-Arrhenius reaction rates

At low T, rates very slow,

but faster than > Arrhenius rate

(curvature from line)

very strong D/H isotope effect:

hallmark of tunneling.